AN ANALYSIS OF MICROWAVE DE-EMBEDDING ERRORS

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ABSTRACT: Bounds on de-embedding errors are derived. Both measurement errors and "known" load errors are considered. Of particular interest is the determination of error bounds for unknown loads situated in regions of the Smith chart that are remote from the location of the known de-embedding loads. An example is presented. This analysis is of particular interest for de-embedding microwave diodes.

INTRODUCTION

In this paper some bounds on de-embedding errors are derived which are especially applicable when measuring microwave diodes. De-embedding is a two-step process. The first step is to unterminate the embedding network N (see Fig. 1) by the use of known or standard loads[1]. The second step is to use the model that we obtain for the embedding network from the first step, together with measurements of the unknown load taken through the network N, to estimate the parameters of the unknown load. Frequently we are constrained to use standard loads that are located in only one region of the Smith chart. For instance, varactors, which may be used for de-embedding[2] are limited to the capacitive region of the Γ_L-plane. We are particularly interested here in deriving the error for unknowns that are located in regions of the Smith chart which are remote from the location of the standard loads. For example, given standard loads located at $\Gamma_L = -1, -j$, and 1, what is the de-embedding error of an unknown load located at $\Gamma_L = 0$ or $\Gamma_L = j$? We shall return to this example.

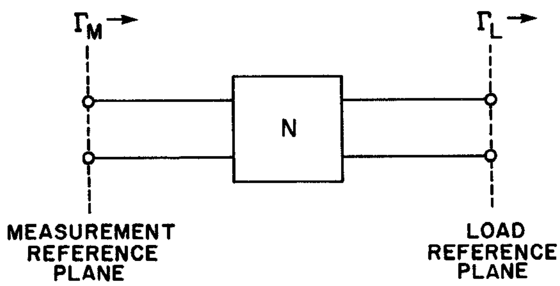

Figure 1 The unknown embedding network N terminated with a known load Γ_L.

We consider two sources of error. δ_{1M}: error in the measurement of the reflection coefficient at the input of network N and δ_{2L}: error in our knowledge of the standard loads. δ_{2L} is the radius of a small circle on the Γ_L-plane within which we expect the reflection coefficient of a standard load, Γ_L, to lie. δ_{2L} is a function of Γ_L. δ_{1M} defines the radius of an uncertainty circle around the measured reflection coefficient Γ_M at the input of network N. δ_{1M} increases with measurement uncertainty. δ_{1M} is a function of the input reflection coefficient, Γ_M; but, since Γ_M is related to Γ_L by a bilinear transformation, δ_{1M} is also a function of Γ_L. In this analysis we assume that δ_{1M} and δ_{2L} have been determined previously either theoretically or experimentally. For instance, in our work on cryosars[3], δ_{2L} was determined by the measurement scatter among the diode packages that hold MOS capacitor standards. In other words, by using highly redundant data, we could obtain a handle on the size of δ_{2L} as a function of Γ_L. δ_{1M} may be caused by noise, nonlinearities, or calibration errors in the reflectometer.

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ERROR ANALYSIS

We determine the scattering parameters of network N, correct to zero order, by the use of a standard unterminating procedure such as that of Bauer and Penfield[1]. From their unterminating calculations we obtain estimates of S_{11}, S_{22}, and Δ where $\Delta \equiv S_{11}S_{22} - S_{12}S_{21}$. We have

$$\Gamma_M \approx \frac{S_{11} - \Gamma_L \Delta}{1 - \Gamma_L S_{22}} . \qquad (1)$$

It is desirable to relate the radius of measurement error, δ_{1M}, at the input reference plane to an equivalent radius of error, δ_{2M}, at the Γ_L reference plane. We know that such a relation exists because bilinear transformations map circles into circles. We may find this relation by taking the variation of Eq. (1).

$$\frac{d\Gamma_M}{d\Gamma_L} = \frac{S_{11}S_{22} - \Delta}{(1 - \Gamma_L S_{22})^2} \qquad (2)$$

or to zero order

$$\delta_{2M} = \left| \frac{(1 - \Gamma_L S_{22})^2}{S_{11}S_{22} - \Delta} \right| \delta_{1M}. \qquad (3)$$

We may define a total equivalent error at the output reference plane as

$$\delta'_{2L}(\Gamma_L) \equiv \delta_{2L}(\Gamma_L) + \delta_{2M}(\Gamma_L). \qquad (4)$$

Since the data used in the unterminating analysis contain errors, the scattering parameters (S_{11}, S_{22}, Δ) computed for the network N, in reality, do not specify N but rather some network model, M. An unknown load Γ_U and its corresponding measured input reflection coefficient Γ_N (which we assume for the moment contains no measurement error) are related by the scattering parameters of N. By invoking the network model M, Γ_U is estimated as Γ'_U. Since both N and M are perfectly well defined, there exists a bilinear transformation between Γ'_U and Γ_U. We may write this transformation as

$$\Gamma'_U = \frac{A + \Gamma_U}{1 - B - C\Gamma_U} . \qquad (5)$$

This is shown schematically in Fig. 2(a). M^{-1} is the network which undoes the bilinear transformation of network M. (In MARTHA[4] parlance $M^{-1} \leftarrow {}^{-}1$ ZSCALE WN M.) Note that by our construction of the problem the magnitude of the *error* in Fig. 2(b) is less than $\delta'_{2L}(\Gamma_L)$. Because of the assumption that S_{11}, S_{22}, and

Δ are correct to zeroth order, $\Gamma_U' \approx \Gamma_U$ and therefore the magnitudes of A, B, and C are much smaller than one. This may be checked by comparing the measured values of Γ_M with those that can be computed by using (1). They should differ in magnitude by no more than $\delta_{1M} + \delta_{1L}$ where δ_{1L} is defined in a manner analogous to δ_{2M} in (3),

$$\delta_{1L} \equiv \left| \frac{S_{11}S_{22} - \Delta}{(1 - \Gamma_L S_{22})^2} \right| \delta_{2L}. \quad (6)$$

The complex de-embedding error $\Delta_{2U}(\Gamma_U)$, which includes only errors from the unterminating procedure, is defined as the difference between the estimated and true values of the load reflection coefficient. Expanding (5) to first order, we obtain

$$\Delta_{2U}(\Gamma_U) \equiv \Gamma'_U - \Gamma_U \approx (1, \Gamma_U, \Gamma_U^2)\ (A,B,C,)^T. \quad (7)$$

We would like to relate the magnitude of Δ_{2U} to the equivalent radii of error $\delta'_{2L}(\Gamma_L)$. This is done by examining $\Delta_{2U}(\Gamma_U)$ at three known loads, that is, $\Delta_{2U}(\Gamma_L)$. Comparing Figs. 2(a) and 2(b) we observe that for the special case of $\Gamma_U = \Gamma_L$ (and therefore $\Gamma_N = \Gamma_M$) we may identify $\Delta_{2U}(\Gamma_L)$ of Fig. 2(a) with the *error* of Fig.2(b). We may state

$$|\Delta_{2U}(\Gamma_L)| \leq \delta'_{2L}(\Gamma_L). \quad (8)$$

To obtain the desired bound on $|\Delta_{2U}(\Gamma_U)|$ we first evaluate (7) at $\Gamma_U = \Gamma_L$. A relation between the parameters A, B, and C, and $\Delta_{2U}(\Gamma_L)$ is found for the three loads under consideration. A bound on $\Delta_{2U}(\Gamma_U)$ may then be found by invoking the triangle inequality.

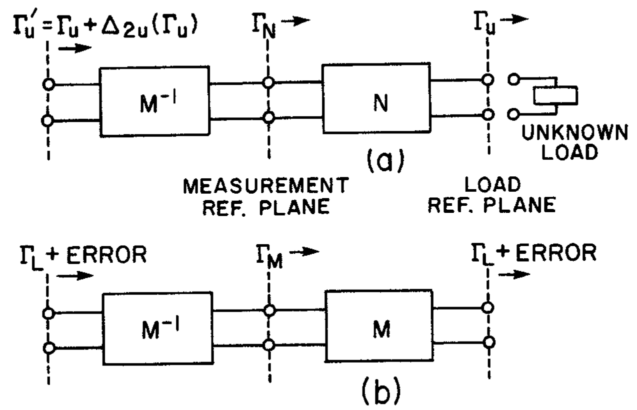

Figure 2 (a) Schematic of how the network model M is used to de-embed an unknown load Γ_U. The resulting error is $\Delta_{2U}(\Gamma_U)$.
(b) A useful network identity from the definition of M. The magnitude of the *error* is less than δ'_{2L} by definition.

AN EXAMPLE

The procedure is best illustrated by an example. Consider, for instance, the three loads $\Gamma_L = 1, -1, -j$. Evaluating (7) for these loads and letting $\alpha_1 \equiv \Delta_{2U}(\Gamma_L = 1)$, $\alpha_2 \equiv \Delta_{2U}(\Gamma_L = -1)$, and $\alpha_3 \equiv \Delta_{2U}(\Gamma_L = -j)$, we obtain

$$(\alpha_1, \alpha_2, \alpha_3) = (A, B, C)H^T$$

where

$$H \equiv \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & -j & -1 \end{pmatrix}. \quad (9)$$

H is a function of the three standard loads. Combining (7) and (8), we get

$$\Delta_{2U} = (1, \Gamma_U, \Gamma_U^2)H^{-1}\ (\alpha_1, \alpha_2, \alpha_3)^T \quad (10)$$

where H^{-1} may be evaluated as

$$H^{-1} = \frac{1}{4} \begin{pmatrix} 1+j & 1-j & 2 \\ 2 & -2 & 0 \\ 1-j & 1+j & -2 \end{pmatrix}. \quad (11)$$

α_1, α_2, and α_3, however, represent the equivalent error of the standard loads that are known in terms of δ'_{2L} from the first part of this article. Therefore we write

$$|\alpha_1| \leq \delta'_{2L}(\Gamma_L = 1);\ |\alpha_2| \leq \delta'_{2L}(\Gamma_L = -1);$$
$$|\alpha_3| \leq \delta'_{2L}(\Gamma_L = -j). \quad (12)$$

By using the triangle inequality we can rewrite (10)

$$|\Delta_{2U}(\Gamma_U)| \leq \frac{1}{4} |1 + j + 2\Gamma_U + \Gamma_U^2 - j\Gamma_U^2|\delta'_{2L}(\Gamma_L = 1)$$
$$+ \frac{1}{4} |1 - j - 2\Gamma_U + \Gamma_U^2 + j\Gamma_U^2|\delta'_{2L}(\Gamma_L = -1)$$
$$+ \frac{1}{4} |2 - 2\Gamma_U^2|\delta'_{2L}(\Gamma_L = -j). \quad (13)$$

It is interesting to evaluate (13) for the two unknown loads mentioned earlier. We also assume $\delta'_{2L}(\Gamma_L = 1) = \delta'_{2L}(\Gamma_L = -j) = \delta'_{2L}(\Gamma_L = -1) \equiv \varepsilon$. Under these conditions,

$$|\Delta_{2U}(\Gamma_U = 0)| \leq 1.2\varepsilon$$
$$|\Delta_{2U}(\Gamma_U = j)| \leq 3\varepsilon. \quad (14)$$

This illustrates clearly how the error grows in regions of the Smith chart that are remote from the location of the standard loads. The total radius of error for Γ_U at the load reference plane is

$$\delta_{TOTAL}(\Gamma_U) = |\Delta_{2U}(\Gamma_U)| + \delta_{2M}. \quad (15)$$

Although it is demonstrated by a specific example, the error analysis outlined here is very general. It can also be programmed easily for computer or calculator use.

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